Piano String Physics

A discussion of Mass-Length-Frequency relationships to Tension, Volume and Inharmonicity

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Agenda

- Review of Relationships
- Gathering String Data
- The Math
- Selecting the Replacement
- Demonstration

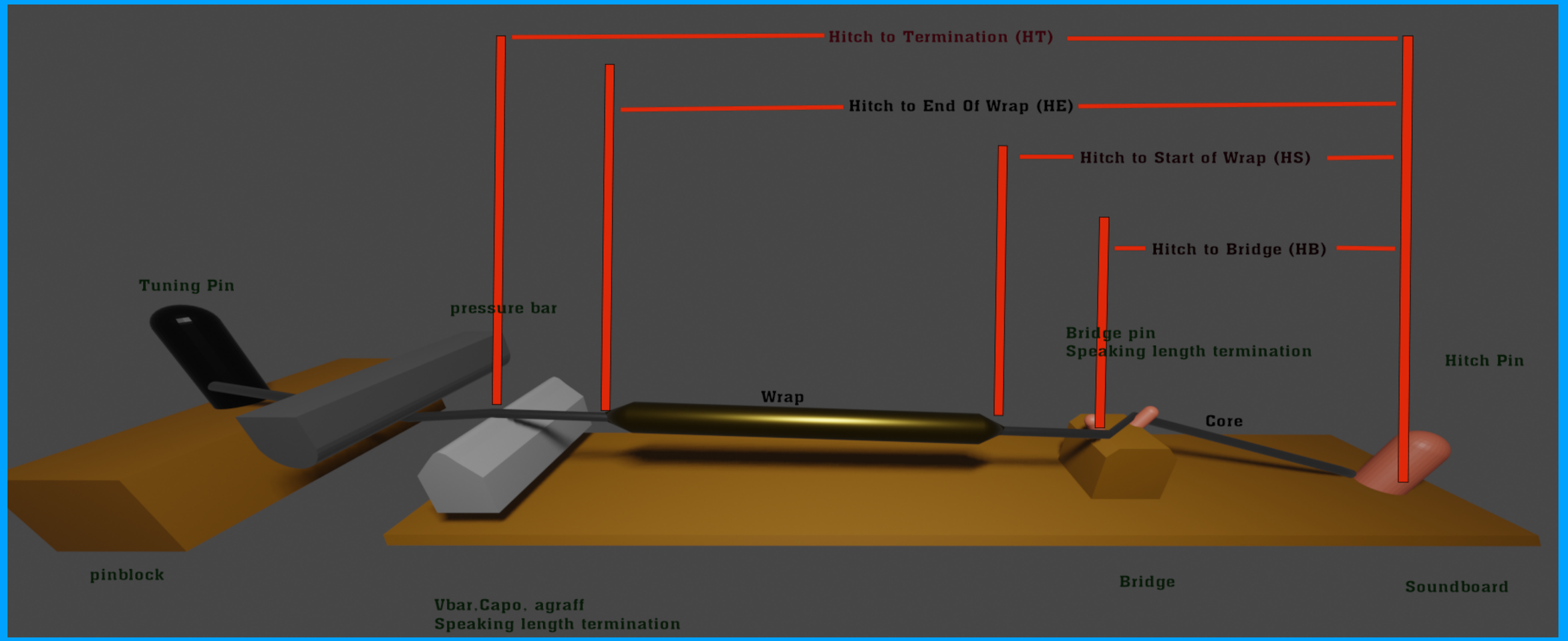
Review of Relationships

How Tension-Volume-Inharmonicity are effected by Length, Mass, Frequency

- Change in Length: L ↑ then T ↑ V ↑ I ↓
- Change in Mass: M ↑ then T ↑ V ↑ I ↓
  - Change in Core: C ↑ then T ↑ V ↑ I ↑
  - Change in Wrap: W ↑ then T ↑ V ↑ I ↓
- Change in Frequency: F ↑ then T ↑ V ↑ I ↑

Gathering String Data

String Components Measuring



The Math

got a calculator?

- Speaking Length = HT - HB
- Bridge = HS - HB
- VBAR = HT - HE
- Core: Diameter of plain wire in MILS
- Wrap: Overall outside Diameter of wrapped part in MILLS

TENSION

$$T = (\exp((nn/6) * \log(2) * (Ls * D)^2 / (GRAIN/F^2)) * ((K * ((d2^2 - D^2) / D^2)) + 1)$$

Where:

nn = Note Number (1-88)

Ls = Speaking Length in Inches

D = Core diameter in MILLS

d2 = Wrap diameter MILLS

K = Specific Gravity of Winding Material

GRAIN = 124710150000 a constant related to Stainless Steel

F = Frequency of the Note (A4 = 440, A0 = 27.5)

Note: GRAIN for demonstration purposes based on A4@440hz for this demonstration

VOLUME

$$V = (\sqrt{(U * T * D^2 * (K * ((d2^2 - D^2) / D^2) + 1))}) / 100$$

Where:

U = number of strings in the unison
T = Tension
D = Core diameter
d2 = Wrap diameter
K = Specific Gravity of Winding Material

INHARMONICITY

If PLAIN wire:

$$I = D^4 / (81 * T * Ls^2)$$

If Single Wound:

$$I + (0.287 * (d2/(1+d2)))*(4*\sin((4*pi*VBAR)/Ls) - \sin((16*pi*VBAR)/Ls) + 4*\sin((4*pi*BRIDGE)/Ls) - \sin((16*pi*BRIDGE)/Ls))$$

If Double Wound:

Single Wound + $0.287*K*d2*IWS/D^2$ *

$$((0.287 * (d2/(1+d2)))*(4*\sin((4*pi*VBAR+Step)/Ls) - \sin((16*pi*VBAR+Step)/Ls) - (0.287 * (d2/(1+d2)))*(4*\sin((4*pi*VBAR)/Ls) - \sin((16*pi*VBAR)/Ls) + (0.287 * (d2/(1+d2)))*(4*\sin((4*pi*BRIDGE+Step)/Ls) - \sin((16*pi*BRIDGE+Step)/Ls) (0.287 * (d2/(1+d2)))*(4*\sin((4*pi*BRIDGE)/Ls) - \sin((16*pi*BRIDGE)/Ls))$$

Where: VBAR = HT - HE, BRIDGE = HS-HB, Step = length go exposed underwrap,
D = Core Diameter in Mills, d2 = Wrap Diameter in Mills,
IWS = underwrap Diameter in Mills, Ls = HT-HB,
K = Specific Gravity of Winding Material

The Easy Way

http://oppor-tune-ist.com/page/StringCalculator/StringCalculatorInput.html

Practical Example

Senario:
An old upright with string for note 7 is no where to be found.

String 6 parameters:
HB = 4.0 (10.3cm), HS = 4.5 (11.4cm), HE = 45.0(114.3), HT = 46.0(116.8)
Core: 40, Wrap: 198, Material: Copper, Note D1

String 8 parameters:
HB = 4.0 (10.3cm), HS = 4.5 (11.4cm), HE = 44.5.0(113.0), HT = 45.5(115.6)
Core: 41, Wrap: 180, Material: Copper, Note E1

What we know or may assume of String 7 parameters:
HB = 4.5(11.4cm), HS = 5.0 (12.7cm), HE = 45.25(114.9), HT = 46.25(117.5)
Core: ?(avg. 40.5, use 41), Wrap: ?(avg. 189), Material: Copper, Note D#1

Analysis

String 6:Tension: 169.1 Volume: 24.3 Inharmonicity: 0.21

String 8:Tension: 172.0 Volume: 22.3 Inharmonicity: 0.23

String 7:Tension: 171.6 Volume: 23.4 Inharmonicity: 0.23

What do you do TODAY?

A demonstration of the future:

Iphone:

Python: